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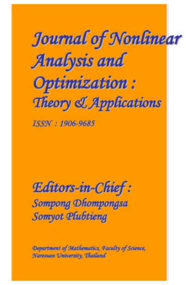

SOME NEW PROPERTIES OF GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT:
In this article, we presented the new class of generalized closed sets called $\tilde{a}$- closed and g$\tilde{a}$-cld which contains the above-mentioned class. We also examine the connections between related generalized closed sets.
Keywords: g$\tilde{a}$-cld, g$\tilde{a}$-cld, gsp-cld, gs-cld, sg-cld.

INTRODUCTION :
As a generalization of closed sets, Levine [11] introduced generalized closed sets in general topology. This concept was seen as beneficial, and numerous results advanced generalized topology as a whole. In topological spaces, ĝ-closed was introduced by numerous researchers, such as Veerakumar [19]. In topological spaces, ω-closed was first introduced by Sheik John [17]. Andrijevic, D [2], introduced the some properties of the topology of □-sets and Arya, S. P. and Gupta, R [4], introduced the on strongly continuous mappings. Following the introduction of these concepts, generalized topology focused on the main characteristics of various types of generalized closed. Carnation, D [6], introduced the some properties related to compactness in topological spaces. Ravi and Ganesan [16] as an additional generalization of closed. They showed that the class of &g-closed appropriately falls between the classes of closed and g-closed. According to Pious Missier et al. [15], introduced by g‴-closed. Noiri, T., Maki, H. and Umehara, J [14], introduced the generalized preclosed functions.
In this article, we presented the new class of generalized closed sets called $\tilde{a}$- closed and g$\tilde{a}$-cld which contains the above-mentioned class. We also examine the connections between related generalized closed sets.

PRELIMINARIES:
Throughout this paper (X, □), (Y, □) and (Z, □) (or X, Y and Z) represent topological spaces (briefly, TPS) on which no separation axioms are assumed unless otherwise mentioned.
We recall the following definitions which are useful in the sequel.

Definition 2.1
A subset T of a space X is said to be:
(i) semi-open [10] if T □ cl(int(T));
(ii) □-open [13] if T □ int(cl(int(T)));
(iii) semi-preopen [1, 7] if T □ cl(int(cl(T)));
(iv) regular open [18] if T = int(cl(T)).

Definition 2.2
A subset T of a space X is said to be:
(i) a generalized closed (briefly g-cld) [11] if cl(T) □ whenever T □ P and P is open.

(ii) a generalized semiclosed (briefly gs-cld)[3, 8] if scl(T) □ P whenever T □ P and P is open.

(iii) a □ -generalized closed (briefly □ g-cld) [12] if □ cl(T) □ P whenever T □ P and P is open.

(iv) a generalized semi-preclosed (briefly gsp-cld) [9] if spcl(T) □ P whenever T □ P and P is open.

(v) a semi-generalized closed (briefly sg-cld) [5] if scl(T) □ P whenever T □ P and P is semi-open.

1. g$\tilde{\alpha}$-CLOSED SETS

We introduce the definition of generalized $\tilde{\alpha}$-closed sets in TPS and study of such sets.

Definition 3.1

A subset T of a TPS is called

(i) a $\tilde{\alpha}$-closed (briefly $\tilde{\alpha}$-cld) if cl(T) □ P whenever T □ P and P is sg-open.

(ii) a generalized $\tilde{\alpha}$-closed (briefly g$\tilde{\alpha}$-cld) if cl(int(T)) □ P whenever T □ P and P is sg-open.

Theorem 3.2

Any closed □ g$\tilde{\alpha}$-cld but reverse is not true.

Proof

Let T be a closed set. Then cl(T)= T. Let T $\subseteq$ P and P be sg-open. Since int(T) $\subseteq$ T, cl(int(T)) $\subseteq$ cl(T) = T. We have cl(int(T)) $\subseteq$ T $\subseteq$ P whenever T $\subseteq$ P and P is sg-open. Hence T is g$\tilde{\alpha}$-cld.

Example 3.3

Let X = {i_1, s_1, d_1} and □ = {□, {i_1}, {s_1}, {i_1, s_1}, X}. Then the set {i_1, s_1} is g$\tilde{\alpha}$-cldset but not closed.

Theorem 3.4

Any $\tilde{\boldsymbol{\alpha}}$-cld □ g$\tilde{\boldsymbol{\alpha}}$-cld but reverse is not true.

Proof

The proof is straight forward.

Example 3.5

In Example 3.3, the set {i_1, s_1} is g$\tilde{\boldsymbol{\alpha}}$-cld set but not $\tilde{\boldsymbol{\alpha}}$-cld in X.

Theorem 3.6

Any regular cld □ g$\tilde{\boldsymbol{\alpha}}$-cld but reverse is not true.

Proof

Let T be any regular cld set and let *B* be gs-open set containing T. Since T isregular cld, we have T = cl(int(T)) □ U. Thus, T is g$\tilde{\boldsymbol{\alpha}}$-cld.

Example 3.7

In Example 3.3, the set {i_1} is g$\tilde{\boldsymbol{\alpha}}$-cld but not regular cld in X.

Theorem 3.8

Any g$\tilde{\boldsymbol{\alpha}}$-cld □ gsp-cld but reverse is not true.

Proof

Let T be any g$\tilde{\boldsymbol{\alpha}}$-cld and *B* be open set containing T. Then *B* is a sg-open containing T and cl(int(T)) □ *B*. Since *B* is open, we get int(cl(int(T))) □ *B* which implies spcl(T) = T □ int(cl(int(T))) □ U. Thus, T is gsp-cld.

Example 3.9

Let X = {i_1, s_1, d_1} and □ = {□, {i_1}, {s_1}, {i_1, s_1}, X}. Then the set {i_1} is gsp-cldbut not g$\tilde{\boldsymbol{\alpha}}$-cld.

Theorem 3.10

If a subset T of a *TPS* X is both closed and □ g-cld, then it is g$\tilde{\boldsymbol{\alpha}}$-cld in X.

Proof

Let T be an □ g-cld set in X and *B* be an open set containing T. Then *B* □□□ cl(T) = T

□ cl(int(cl(T))). Since T is closed, *B* □ cl(int(T)) and hence T is g$\tilde{a}$-cldin X.

Theorem 3.11

If a subset T of a *TPS* X is both open and g$\tilde{a}$-cld, then it is closed.

Proof

Since T is both open and g$\tilde{a}$-cld, T □ cl(int(T)) = cl(T) and hence T is closed in X.

Corollary 3.12

If a subset T of a *TPS* X is both open and g$\tilde{a}$-cld, then it is both regular openand regular cld in X.

Theorem 3.13

A set T is w$\tilde{a}$-cld if and only if cl(int(T)) □ T contains no non-empty gs-cld.

Proof

Necessity. Let F be a gs-cld such that F □ cl(int(T)) □ T. Since F^c is gs-open and T □ F^c, from the definition of w$\tilde{a}$-cld it follows that cl(int(T)) □ F^c. ie. F □ $(cl(int(T)))^c$. This implies that F □ (cl(int(T))) □ $(cl(int(T)))^c$ = □.

Sufficiency. Let T □ G, where G is both closed and sg-open set in X. If cl(int(T)) is not contained in G, then cl(int(T)) □ G^c is a non-empty gs-closed subsetof cl(int(T)) □ T, we obtain a contradiction. This proves the sufficiency and hence the theorem.

Theorem 3.14

Let X be a *TPS* and T □ Y □ X. If T is open and g$\tilde{a}$-cld in X, then T is w$\tilde{a}$- cld relative to Y.

Proof

Let T □ Y □ G where G is gs-open in X. Since T is g$\tilde{a}$-cld in X, T □ G implies cl(int(T)) □ G. That is Y □ (cl(int(T))) □ Y □ G where Y □ cl(int(T)) is closure of interior of T in Y. Thus, T is g$\tilde{a}$-cld relative to Y.

Theorem 3.15

If a subset T of a *TPS* X is nowhere dense, then it is g$\tilde{a}$-cld.

Proof

Since int(T) □ int(cl(T)) and T is nowhere dense, int(T) = □. Therefore cl(int(T)) = □ and hence T is g$\tilde{a}$-cld in X.

The converse of Theorem 3.15 need not be true as seen in the followingexample.

Example 3.16

Let X = $\{i_1, s_1, d_1\}$ and □ = {□, $\{i_1\}$, $\{s_1, d_1\}$, X}. Then the set $\{i_1\}$ is g$\tilde{a}$-cld but not nowhere dense in X.

Remark 3.17

The following examples show that g$\tilde{a}$-cld and semi-closedness are independent.

Example 3.18

In Example 3.16, the set $\{s_1\}$ is g$\tilde{a}$-cld but not semi-cld in X.

Example 3.19

Let X = $\{i_1, s_1, d_1\}$ and □ = {□, $\{i_1\}$, $\{s_1\}$, $\{i_1, s_1\}$, X}. Then the set $\{s_1\}$ is semi-closed set but not g$\tilde{a}$-cld in X.

Definition 3.20

A subset T of a *TPS* X is called g$\tilde{a}$-open set if A^c is g$\tilde{a}$-cld in X.

Theorem 3.21

Any open set □ g$\tilde{a}$-open.

Proof

Let T be an open set in a *TPS* X. Then T^c is closed in X. By Theorem 3.2 itfollows that T^c is g$\tilde{a}$-cld in X. Hence T is g$\tilde{a}$-open in X.

The converse of Theorem 3.21 need not be true as seen in the followingexample.

Example 3.22

In Example 3.3, the set $\{d_1\}$ is g$\tilde{a}$-open set but it is not open in X.

Proposition 3.23

(i) Any ***ã***-open set □ g***ã***-open but reverse is not true.
(ii) Any regular open □ g***ã***-open but reverse is not true.
(iii) Any g-open set □ g***ã***-open but reverse is not true.
(iv) Any g***ã***-open set □ gsp-open but reverse is not true.

It can be shown that the converse of (i), (ii), (iii) and (iv) need not be true.

Theorem 3.24

A subset T of a *TPS* X is g***ã***-open if G □ int(cl(T)) whenever G □ T and G is gs-cld.

Proof

Let T be any g***ã***-open. Then T^c is g***ã***-cld. Let G be a sg-cld contained in T.Then G^c is a sg-open set containing T^c. Since T^c is g***ã***-cld, we have cl(int(T^c)) □ G^c.Therefore G □ int(cl(T)).Conversely, we suppose that G □ int(cl(T)) whenever G □ T and G is sg- closed. Then G^c is a sg-open set containing T^c and G^c □ $(\text{int}(\text{cl}(T)))^c$. It follows that$G^c$ □ cl(int(T^c)). Hence T^c is g***ã***-cld and so T is g***ã***-open.

REFERENCES :

[1] Andrijevic, D.: Semi-preopen sets, Mat. Vesnik, 38 (1986), 24-32.
[2] Andrijevic, D.: Some properties of the topology of □-sets, Mat. Vesnik, 36 (1984), 1-10.
[3] Arya, S. P. and Nour, T. M.: Characterizations of s-normal spaces, Indian J. Pure Appl. Math., 21 (1990), 717-719.
[4] Arya, S. P. and Gupta, R.: On strongly continuous mappings, Kyungpook Math. J., 14 (1974), 131-143.
[5] Bhattacharya, P. and Lahiri, B. K.: Semi-generalized closed sets in topology, Indian J. Math., 29(3) (1987), 375-382.
[6] Carnation, D.: Some properties related to compactness in topological spaces, Ph. D. Thesis, University of Arkansas, 1977.
[7] Crossley, S. G. and Hildebrand, S. K.: Semi-closure, Texas J. Sci., 22 (1971), 99-112.
[8] Devi, R.: Studies on generalizations of closed maps and homeomorphisms in topological spaces, Ph. D Thesis, Bharathiar University, Coimbatore (1994).
[9] Dontchev, J.: On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 16 (1995), 35-48.
[10] Levine N.: Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
[11] Levine N.: Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(1970), 89-96.
[12] Maki, H., Devi, R. and Balachandran, K.: Associated topologies of generalized □ -closed sets and □ -generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 15 (1994), 51-63.
[13] Njastad, O.: On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961-970.
[14] Noiri, T., Maki, H. and Umehara, J.: Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. Math., 19 (1998), 13-20.
[15] Pious Missier, S., Ravi, O. and. Herin Wise Bell, P.: *g*□□□ -closed sets in topology (communicated).
[16] Ravi, O. and Ganesan, S.: $\ddot{g}$-closed sets in topology, International Journal of Computer Science and Emerging Technologies, 2(3) (2011), 330-337.
[17] Sheik John, M.: A study on generalizations of closed sets and continuous maps in topological and bitopological spaces, Ph. D Thesis, Bharathiar University, Coimbatore, September 2002.
[18] Stone, M. H.: Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937), 374-481.
[19] Veera Kumar M. K. R. S.: $\hat{g}$ □ closed sets in topological spaces,